DISORIENTED CHIRAL CONDENSATES: A DYNAMICAL SIMULATION IN THE (2+1)-DIMENSIONAL GROSS-NEVEU MODEL *

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Abstract

We simulate the formation and growth of disoriented chiral condensate (DCC) regions which follow the expansion of a high energy density region into the "cold" vacuum. The numerical study is based on the one-loop effective potential for the massive 2+1-dimensional Gross-Neveu model. We pay attention to the setting of the initial conditions and to determining which parameters are relevant for a strong amplification of the pion field. We find that the size of the "hot" source plays a significant role. For large enough source radii, we observe strong correlation phenomena, corresponding to the growth of large regions where the pion field oscillates along a given direction. We give our results in terms of the θ angle which defines the DCC disorientation, of the other $O(4)$ angles distributions, of the local ratios $R_a = \pi_a^2/\vec{\pi}^2$ $(a = 1, 2, 3)$, and of the energies associated with the fields at representative times.

1. Introduction

It is well known that QCD has an approximate $SU(2)_L \otimes SU(2)_R$ global chiral symmetry and that the pions are the pseudo-goldstone bosons associated to the spontaneous breaking of the symmetry down to $SU(2)_{L+R}$. On the other hand, asymptotic freedom implies that there should be qualitative changes in the properties of hadronic matter as the temperature and/or the baryonic density are increased. Thus we expect a colour deconfining phase transition. Also another phenomenon is expected, which is just the restoration of chiral symmetry. There are indications that these phase transitions occur at the same critical temperature, presumably located around 150 MeV [1]. Since such temperatures are supposed to be attainable in high-energy collisions of heavy ions, much effort has already been done to identify possible experimental signatures of the phase transition, primarily of the deconfinement[2].

One very interesting proposal on how to decide whether the restoration of chiral symmetry was reached or not during a heavy ion collision was suggested during the last few years [3, 4, 5, 6, 7]. The basic mechanism is the following: closely above the chiral critical temperature T_c, chiral symmetry implies the equivalence of the different orientations in the internal space. The non-equilibrium expansion and consequent cooling of the plasma makes however the chiral symmetry to break and the direction of the symmetry breaking in the internal space does not necessarily have to be the same as it originally was before the collision took place. The small chiral breaking terms are presumably negligible at this stage.

To explain the possible physical consequences, the language of the linear σ-model [8] has been widely employed, since the relevant part of the order parameter is a four-component vector $\phi^\alpha \equiv (\sigma, \vec{\pi})$, and the chiral transformations are $O(4)$ rotations in the internal space. In terms of these fields, the usual zero-temperature vacuum is defined by $\langle\sigma\rangle = f_\pi$ and $\langle\vec{\pi}\rangle = 0$. Thus, the situation depicted above, indicates the possibility that before the plasma approaches again equilibrium, in some region of space-time the vacuum orientation can be different from the usual one, namely: $\langle\sigma\rangle = f_\pi \cos\theta$ and $\langle\vec{\pi}\rangle = f_\pi \vec{n} \sin\theta$, where $\vec{n}$ is a some unity vector in the internal $O(3)$ pion space. Because of the fact that the pion mass is small, it costs relatively little energy, of the order of $f_\pi^2 m_\pi^2$ per unit volume, to disorient the vacuum. Macroscopic domains containing a quasi-classical pion field can appear as the temperature drops below T_c [6, 7].

Such metastable domains will eventually disappear as the system evolves towards the true vacuum, mainly via $O(3)$-coherent emission of pions. The formation of large domains could lead to events with an unusually large number of either neutral π^0 or charged $\pi^\pm$ pions (Centauro and anti-Centauro events [9]).

The scenario that we have summarized is referred to in literature as Disoriented Chiral

Condensates (DCC) [4, 5, 6].

A specific prediction for the distributions of the fraction of neutral pions emerging from each domain can be obtained from the following semiclassical arguments (see e.g. [6, 7] and refs. therein). Let us consider a single (ideal) domain in which the pion field is oscillating along a fixed direction $\vec{n}$. Classically, pion production from this domain is proportional to the square of the field strength (think of non-relativistic coherent states), so that the fraction of the neutral pion is $R = \pi_0^2/\vec{\pi}^2$. Since all orientations of $\vec{\pi}$ are a priori equally likely, the probability of finding a fraction R of neutral pions is $P(R) = 1/(2\sqrt{R})$ (see for instance [7]). In the case of events dominated by a single large domain, this feature would thus reflect on the distribution probability[1] of the observed fraction of neutral pions, whose shape strongly deviates from the binomial distribution of typical hadronic collisions. We remark that the number of soft pions produced at high energies in such collisions could be quite large, which makes plausible the validity of semiclassical arguments. In fact, it seems possible that at high energy density the produced pions can create a certain coherent state, i.e. a classical pion field [3].

In order to study these problems one should consider, strictly speaking, the real hadron world (quarks and gluons) with all its complexities. Unfortunately, the detailed dynamics of heavy ion physics is far beyond what can be calculated directly from QCD. Therefore most discussions of DCC are necessarily based on simplified models which can describe the low energy behaviour of QCD.

Actually in recent literature, the effective Lagrangian of the $O(4)$ linear σ-model has been used to show, by numerical simulations, the amplification of soft pion modes following a quench from high to low temperatures [7]. Other numerical studies have incorporated the feature of the expansion of the plasma by starting from a spatially non uniform energy distribution, and have progressed also in discussing the role of the fluctuations [10].

Other interesting studies on the subject can be found in ref. [11, 12, 13].

In this work we present a numerical study, performed on the basis of the one-loop effective potential of the Gross-Neveu model in 2+1 dimensions, which we believe can retain in these connections some aspects of real QCD [14].

Let us briefly comment on the choice of this model. Though less physical, a two-space-dimensional model allows for faster computer simulations, thus opening the possibility of a more thorough analysis. Some main features of the Gross-Neveu model in 2+1 dimensions are known, even if in an equilibrium context and in the mean field approximation [15]. Thus, in principle, one could compare some equilibrium property (for instance, in order to better discuss the initial conditions of a quench), as derived from a microcanonical simu-

[1] Probability on a event-by-event basis, i.e. a distribution over many events.

lation and from the canonical-ensemble study of ref.[15]. Furthermore, the one-loop zero temperature effective potential of the model, expressed in terms of the appropriate scalar and pseudoscalar composite fields, comes directly from the integration over fundamental fermion fields of the underlying theory, and has a very simple expression sharing the principal features of the σ-model. Incidentally, the form of the potential raises the problem of better exploring the effects of different kind of non-linearities.

Coming back to the phenomenology of DCC, we simulate the amplification of the pion field following a quench, i.e. a sudden transition from high to low temperature (from T above T_c to T below T_c). In our case the quench is realized by means of the expansion of a high-energy-density region (in which the field fluctuates around the maximum of the double-well potential), into the cold vacuum surrounding it (in which the field performs small-amplitudes fluctuations around the absolute minimum of the potential) [10].

In such a simulation, the only "thermal" character of the system lies in the initial conditions. Consistently, we pay attention to controlling the role of the parameters which describe our initial conditions, such as the energy density, the source radius etc., and to build up a simulation scheme suited to give the results in terms of physically measurable quantities, such as the energies of the outgoing pions and their number. For the time being, we do not worry too much about a physically quantitative description which will be hopefully gained only by repeating the study in a more realistic 3+1 dimensional model; here we concentrate rather on the observation of the basic mechanisms leading to the amplification of the pion field, as a guidebook to other simulations.

In Sect.2 we summarize the basic features of the Gross-Neveu model and of the effective action we use. In Sect.3 we present and discuss in more detail the numerical simulation. This section is divided in subsections, containing a discussion of the initial conditions, the analysis of the different quantities we have measured, and the sensitivity of the results to changes in the parameters.

In Sect.4 we summarize the results in the conclusions.

2. The Model

In order to study the possible formation of DCC, and its phenomenological consequences, we have carried out numerical simulations for the (2+1)-dimensional massive Gross-Neveu model [14, 15].

The model is defined by a four-fermion interaction Lagrangian density

$$\mathcal{L} = \bar{\psi}\left(i\hat{\partial} - M\right)\psi + \frac{g^2}{2}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\vec{\tau}\gamma_5\psi)^2\right] \tag{2.1}$$

where M is the bare fermion mass, τ_i are the matrices of the fundamental representation of $SU(N_f)$, and ψ consist of N separate N_f-dimensional representation of $SU(N_f)$. In the following we take $N_f = 2$.

For $M = 0$ the model has an exact chiral $SU(2)_L \otimes SU(2)_R$ symmetry which is dynamically broken to $SU(2)_V$. It is renormalizable in the $1/N$ expansion [15, 16].

The four-fermion interaction can be conveniently studied in the $1/N$ expansion by introducing two auxiliary fields $(\sigma, \vec{\pi})$ which satisfy the classical equations of motion

$$\begin{aligned} \sigma &= g\bar{\psi}\psi - \frac{M}{g} \\ \vec{\pi} &= g\bar{\psi}i\gamma_5\vec{\tau}\psi \end{aligned} \tag{2.2}$$

Then the Lagrangian becomes

$$\mathcal{L} = \bar{\psi}i\hat{\partial}\psi - \frac{1}{2}(\sigma^2 + \vec{\pi}^2) + g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi - M\frac{\sigma}{g} \tag{2.3}$$

and by integrating over the fermion fields one can study the effective action as a functional of $(\sigma, \vec{\pi})$.

The effective Lagrangian obtained by this procedure is

$$\mathcal{L}_{eff} = \frac{1}{2c_0^2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\right) - \mathcal{V}_{eff} \tag{2.4}$$

where derivative terms have been kept only up to second order. The one-loop effective potential $\mathcal{V}_{eff}$ is the sum of a chirally symmetric part [15] and a breaking term proportional to σ

$$\mathcal{V}_{eff}(\sigma^2 + \vec{\pi}^2, \sigma) = \frac{N_f N g^3}{2\pi}\left(\frac{(\sigma^2 + \vec{\pi}^2)^{3/2}}{3} - \frac{m_0}{g}\frac{(\sigma^2 + \vec{\pi}^2)}{2}\right) + \frac{M\sigma}{g} \tag{2.5}$$

m_0 being the dynamical mass acquired by the fermions due to the spontaneous breaking of chiral symmetry [2]

[2]The finite temperature extension of this effective potential for $M = 0$ in the Canonical Ensemble has been calculated and discussed in ref.[15]. The mean field calculation shows the existence of a second order phase transition at $T_c = m_0/2\ln 2$, whilst by taking into account the field fluctuations, this transition turns out to be of the Kosterlitz-Thouless type (see refs.[15, 17] and references therein).

Clearly, for dimensional reasons, the σ and $\vec{\pi}$ fields cannot be identified with the canonical ones. The c_0 factor in eq.(2.4), which rescales the fields to the canonical ones $(\varphi_\sigma, \vec{\varphi}_\pi) \equiv (\sigma, \vec{\pi})/c_0$, can be determined by requiring that the effective potential $\mathcal{V}_{eff}$ gives the correct σ-mass, $m_\sigma^2 = 4m_0^2$ [15, 17]. The value we get is $c_0^2 = 8\pi m_0/(N_f N g^2)$.

After some algebra, the effective Lagrangian can be written as

$$\frac{8\pi}{N_f N m_0^3}\mathcal{L}_{eff} = \frac{1}{2m_0^2}\partial_\mu\phi_\alpha\partial^\mu\phi_\alpha - U(\phi_\alpha) \tag{2.6}$$

$$U(\phi_\alpha) = \frac{4}{3}(\phi_\alpha\phi^\alpha)^{3/2} \; - \; 2\phi_\alpha\phi^\alpha \; + \; \lambda\phi^\alpha\delta_{\alpha 1} \tag{2.7}$$

where we have defined

$$\phi_\alpha \equiv \frac{g}{m_0}(\sigma, \vec{\pi}) \qquad \lambda \equiv \frac{8\pi M}{N_f N g^2 m_0^2} \tag{2.8}$$

In Fig.1 we plot the effective potential (2.7) $U(\phi_1, \vec{0}) - U(v, \vec{0})$. Its extrema are located at

$$\begin{aligned} v &= -\frac{1}{2}\left(1 + \sqrt{1+\lambda}\right); && \textit{abs. min.} \\ \phi_0 &= \frac{1}{2}\left(1 - \sqrt{1-\lambda}\right); && \textit{rel. max.} \\ v_+ &= \frac{1}{2}\left(1 + \sqrt{1-\lambda}\right); && \textit{rel. min.} \end{aligned} \tag{2.9}$$

We have set the parameters to $m_0 = 300MeV$ and $\lambda = 1/4$, leading to $m_\sigma = 635MeV$ and $m_\pi = 145MeV$.

For the numerical simulation we employ dimensionless variables

$$(\vec{\xi}, \tau) = (m_0\vec{x}, m_0 t) \tag{2.10}$$

and, from eq.(2.6), the Hamiltonian

$$H \equiv \frac{8\pi}{N_f N m_0^3}\mathcal{H}_{eff} = \sum_{\alpha=1}^{4}\frac{\dot{\phi}_\alpha^2}{2} + \sum_{\alpha=1}^{4}\frac{(\nabla\phi_\alpha)^2}{2} + U(\phi_\alpha) - U(v, \vec{0}) \tag{2.11}$$

where the derivatives are now taken with respect to $(\vec{\xi}, \tau)$, and the vacuum energy has been subtracted.

3. Dynamical Simulation

Formulation

Our aim is to simulate, within the model, the dynamics of the expansion of a high energy density region in a low energy density environment. The first region should simulate the hot fireball after a central collision and the consequent compression, whereas the second region is the usual vacuum.

The central assumption is that, before the fireball begins to expand into the vacuum, it can be viewed as an isolated system close to thermal equilibrium near the critical temperature[3], in a region of linear size of the order of some fm.

In view of the initial conditions we want to represent, the configuration of the σ field must be chosen in a way that the mean value $\langle\sigma\rangle$ is close to zero[4]. This is achieved, in the central region, by assigning to the σ-field a constant value corresponding to the relative maximum of the potential (2.7). The same we do for the $\vec{\pi}$ field, with the difference that the initial configuration is now strictly zero.

To start the dynamics, we assign initial velocities according to a random gaussian distribution with a given variance, the same for the σ and $\vec{\pi}$ fields, as they have to be degenerate in the source (apart for the symmetry breaking term). We remark that, since the mean value of the initial velocities is taken as vanishing, to assign the velocity variance corresponds to determine the first term of the Hamiltonian of eq.(2.11).

Out of the central region, the σ-field should smoothly connect with the cold vacuum where the velocity fluctuations are small, due to the smallness of the energy density. Within the source, the $\vec{\pi}$-field initial amplitude is set constant and equal to zero.

The intermediate region, which connects the "hot" vacuum to the "cold" one, is the most energetic and the energy is carried almost entirely by the σ-field. In fact the gradient term of the Hamiltonian (2.11) grows significantly due to the spatial variation of σ from the edge of the central region to the vacuum outside. Such a growth overcomes the decrease of potential energy if the variation of the field is steep enough in the connecting region.

Finally, in the third region, the "cold" vacuum is represented by putting the σ at the absolute minimum of the potential (2.7) and the pion field to zero, and giving to them a nearly vanishing velocity variance.

The successive evolution is governed by the Hamiltonian of eq.(2.11) and the dynamics soon forces the system to have certain (fixed) ratios between the three terms in the r.h.s. of eq.(2.11), with the velocity fluctuations inducing fluctuations also in the field amplitudes.

[3] As far as the average value of some observables is concerned.

[4] It is not strictly vanishing due to the presence of an explicit symmetry-breaking term in the Lagrangian (2.1)

We note that a nearly constant initial field amplitude provides an easy way to fix the energy at the beginning, since it decouples the Hamiltonian and makes then easier to keep the various terms under control: the successive dynamical evolution then produces a non-constant amplitude, but the energy remains fixed. Moreover, since we are to approximate a smooth field on the lattice, the field values must not vary too much within a lattice spacing, otherwise the field would not be differentiable in the continuum limit and the derivative terms would diverge. Thus we find incorrect to choose an initial field configuration distributed randomly over a length scale of the order of the lattice spacing, unless of nearly vanishing amplitudes. For the same reason the initial velocity variance (and thus the initial energy) cannot grow indefinitely in this scheme.

Let us remark that the gradient term is related to the fluctuations of the field, but it contains more information since it depends also on their spatial distribution.

As far as the numerical simulation is concerned, we solve the classical equations of motion for the dynamics of the fields $\phi_\alpha(x, y, t)$ by using laboratory space-time coordinates. The Hamilton equations are integrated by means of a second order bilateral symplectic algorithm (for the details of the method see ref. [18], whereas a sketch of the method is in the Appendix) on a 200×200 square lattice, with a time integration step $dt = a/10$ where a is the lattice spacing, which we fix to $0.1 fm$ [5].

Initial conditions

The standard initial conditions that we choose are given by setting the following field configuration

$$\phi(\vec{\xi}, \tau = 0) = \begin{pmatrix} v + f(\rho)(\phi_0 - v) \\ \vec{0} \end{pmatrix} \tag{3.1}$$

where $(v, \vec{0})$ and $(\phi_0, \vec{0})$ are respectively the absolute minimum and the relative maximum of $U(\phi_\alpha)$ in eq.(2.7), with values given by eqs. (2.9) ($\lambda = 1/4$). The function $f(\rho)$ is suited to make $\phi_1 = \phi_0$ in a central disk of radius ten sites, and to connect to $\phi_1 = v$ in about five more sites. This corresponds to have the source containing the "hot vacuum" in a disk of radius 1 fm, separated from the "cold vacuum" by a ring of thickness of about 0.5 fm.

[5] We have verified that reducing the temporal step and the lattice spacing does not change our results. The simulation time is limited by the lattice dimensions, since the expansion can be simulated until the energy of the source reaches the lattice boundary.

These features are realized by means of the following $f(\rho)$

$$f(\rho) = \begin{cases} 1 & \rho < \ell \\ \exp\left\{\frac{1}{s}\ \left[\frac{1}{(\rho-\ell)^2 - w^2}\ +\ \frac{1}{w^2}\right]\right\} & \ell < \rho < \ell + w \\ 0 & \rho > \ell + w \end{cases} \tag{3.2}$$

where $\ell = 10$, $s = 1/900$, $w = 10$, $\rho = r/a$, r being the distance from the lattice center.

As it is seen from eq.(3.2), the tail of the distribution extends over a radius of 2 fm, nevertheless $f(\rho)$ is sensibly different from zero only for $\rho a < 1.5\ fm$. An aspect related to the length of the tail is discussed later on.

The initial velocities are given with $\langle \dot{\phi}_\alpha \rangle = 0$ and variance $\delta\dot{\phi}_\alpha^2 \equiv \langle \dot{\phi}_\alpha^2 \rangle$, with values within $[0.01, 1]$. The velocity variance itself is then spatially modulated consistently with the condition of having an high-energy-density and a low-energy-density region. Furthermore, in order to produce small field fluctuations in the "cold" vacuum, we modulate $\delta\dot{\phi}_\alpha^2$ with the function $0.99\ f(\rho) + 0.01$.

In Figs.2, 3 and 4 we plot respectively the configurations on the lattice of the σ-field, of the π_2-field, and that of the energy density, at $t = 0.02fm$ (one algorithm step).

The results of the simulation are shown in the following subsections.

Disorientation of the chiral condensate

To describe our numerical results, it is convenient to consider the following parameterization of the ϕ vector in terms of the $O(4)$ angles

$$(\sigma, \pi_1, \pi_2, \pi_3) = \rho\ (\cos\theta,\ \sin\theta\ \cos\varphi,\ \sin\theta\ \sin\varphi\ \cos\eta,\ \sin\theta\ \sin\varphi\ \sin\eta) \tag{3.3}$$

As anticipated in the introduction, the amplification of the pion field following the quench becomes an important phenomenon only when the θ angle undergoes strong and coherent disorientations over large enough regions, corresponding to a big transfer of energy to the pion field itself. On the other hand, in order to have a visible effect on the produced pions, it is crucial that this transfer of energy takes place in a preferred direction in the internal $O(3)$ space, which is signaled by the distribution of the φ and η angles.

These parameters are important expecially before the decoupling of the pions fields, i.e. until when the θ angle performs small amplitude oscillations around its equilibrium value $\theta_{eq} = \pi$. Actually, it is evident that after this time interval the more natural description is given directly in terms of energy and number of the outgoing pions.

In Fig.5, some θ configurations are shown for representative times. In this picture we plot the θ angle on the internal 32×32 square lattice centered in the source (which initially is a circle of radius $\rho \simeq 10$ sites), by dividing the range $[0, \pi]$ in five equally spaced

regions. Such regions are colored by using a gray scale, ranging from black ($\theta \simeq 0$) to white ($\theta \simeq \pi$).

According to Fig.5 we can distinguish three different behaviours. At early times there is a formation and growth of θ correlated regions inside the source. Such regions tend to melt together in the bulk, while the outer regions align to $\theta = \pi$. This phenomenon ends up at about $t = 1fm$ (see Fig.6), when the whole source is aligned with the true vacuum. Anyway, the system has not relaxed to the true vacuum yet. From the center of the source, in fact, a collective motion starts, which brings, at $t \simeq 2.5fm$, the whole source to be at $\theta = 0$. Afterwards, the system begins to perform collective damped oscillations along the bottom of the potential well[6], while transferring energy to the pion modes. In this event, most of the energy is acquired by the π_2 field, and this happens between $t \simeq 4.5fm$ and $t \simeq 7fm$. At later times, $t \gtrsim 10fm$, the system eventually decouples (see later).

In Fig.6*a* we plot the mean value $\langle\theta(r)\rangle(t)$ (where the mean value is taken on concentric rings of thickness $0.4fm$) to point out the time scales characterizing the different stages of the phenomenon of DCC. In Fig.6*b* we further point out (from Fig.6*a*) the profiles of $\langle\theta(\bar{r})\rangle(t)$ with $0 < \bar{r} < 0.4\ fm$ and $0.8 < \bar{r} < 1.2\ fm$.

In Fig.7 we show another representation which is less advantageous as far as the growth of coherent regions is concerned, but which better takes into account the small θ disorientations which are not visible in Fig.5. Thus in Fig.7 the θ field on the (x, y) lattice is directly given by the orientations of the arrows. These pictures are taken in coincidence of some maxima of disorientation (see Fig.6). As in Fig.5, it can be noticed that the disorientation is spatially incoherent at $t = 0.02fm$ (Fig.7*a*), whereas it has become coherent at $t = 2fm$ and $t = 5fm$ (Fig.7*b*-*c*). Finally, in Fig.7*d*, at $t = 9fm$, the presence of concentric regions of different θ orientation is evident (see also Fig.6*a* by taking a profile at fixed t).

Figs.5,6 and 7 have all been obtained with $\delta\dot{\phi}_\alpha^2 = 0.1$. The qualitative picture does not change by varying the initial velocity variance (and thus the initial energy), whereas the coherence of the phenomenon reduces by raising $\delta\dot{\phi}_\alpha^2$ (in the interval we have considered, i.e. $[0.01, 1]$), due to the consequent increase of the fields fluctuations.

$O(3)$-coherence

The dynamical evolution of the θ angle, that we have sketched in the previous paragraph, reminds us a laser-like mechanism. In fact, first most of the available energy is stored in an unstable ground state, and later, when this state relaxes to the true vacuum, there is a transfer of energy to the pion modes.

[6] Notice, for instance, that the system at $t = 4fm$ and $t = 6fm$ is aligned to $\theta = \pi$, which is represented by the white color in the pictures.

The important question now, is whether this energy is coherently transferred to a single pion field over a large region, or not. In the former case this could lead to a distribution probability of the emitted pions which deviates from the usual binomial one, as anticipated in the introduction (see for example [6, 7, 12, 13]).

To answer this question, and to better analyse the mechanism by which pion collective modes get enhanced, we have proceeded in two different ways. First we have looked, at fixed times, at the distributions of the representative points of the lattice sites in the (φ, η) plane (see Fig.8) The existence of preferred directions in which the pion fields oscillate is signaled by the formations of clusters of points.

These pictures are particularly useful to signal the existence of preferred oscillation directions, but they do not give any information on the spatial correlation of such phenomenon. Thus, to improve the description, and to analyse the formation and growth of spatially correlated regions, we have considered the time evolution of the ratios

$$R_a = \pi_a^2/\vec{\pi}^2 \tag{3.4}$$

$(a = 1, 2, 3)$ at each site of the lattice. These ratios define the orientation of the oscillation direction with respect to the a-th field.

In the real situation, one is especially interested in the ratio $R_0 \equiv \pi_0^2/\vec{\pi}^2$. Anyway, since in these simulations we do not implement the electromagnetic coupling, all the parameterization, and thus each of the three ratios, are a priori equivalent.

Obviously none of these ratios is defined for vanishing pion fields amplitudes and thus, in particular, they are not defined at the time $t = 0$, with our initial conditions. But as the dynamics starts, the pion field begins to perform oscillations in some direction of the internal $O(3)$ space and thus the ratios R_a are perfectly defined. As the initial velocities are given randomly, the resulting distribution is $P(R) = 1/(2\sqrt{R})$, which is a consequence of the initial equal likelihood of any $O(3)$ direction at each site. To get a pictorial representation of the R_a-regions, we have (see for instance [7]) divided the interval of definition of R in three equally probable regions: black if $R \in [0, 1/9]$, grey if $R \in [1/9, 4/9]$ and white if $R \in [4/9, 1]$. Since the expected mean value is $\langle R \rangle = 1/3$, black and white regions represent regions of big deviations from the mean value 1/3. In Fig.9a we show the spatial distribution of R_2 at the initial time $t = 0.02fm$. The equality of the areas and the mean value 1/3 are very well respected. Anyway, the ratios $R_a(x, y)$ and the mean values $\langle R_a \rangle$ are not conserved quantities[7], and thus the equality of the areas has not to be preserved during the evolution of the system. This is strictly related to what we are looking for: the formation of few large regions where the pion field oscillates coherently around a particular direction.

[7] The conserved Noether currents are the isospin ones.

In the other pictures of Fig.9, we show the configuration of the ratio R_2 over the lattice at representative times. We see that, as already mentioned, at $t \simeq 4fm$ a pion collective motion begins. This leads to the formation of large R correlated regions, where the pion field points to a unique direction in the $O(3)$ space (see Fig.8). The plots in Figs.8, 9 refer to the same event of Figs.5, 7 and thus are for $\delta\dot{\phi}_\alpha^2 = 0.1$. The same considerations done at the end of the last subsection hold for what concerns modifications of the initial velocity variance.

Energy of the fields

In order to study the decoupling of the fields, let us expand the effective potential of eq.(2.7) around the absolute minimum $\hat{\phi}^\alpha \equiv (v, \vec{0})$. The excitations of the fields are $(\chi, \vec{\phi}) = \phi^\alpha - \hat{\phi}^\alpha$ (thus χ is proportional to the fluctuations of the σ field, whereas $\vec{\phi} \propto \vec{\pi}$ see eq.(2.8)).

Then, by a straightforward calculation, the quadratic part of the excitation energy density at each site can be separated from the interaction term

$$\begin{aligned}
E_\chi &= \sum_{sites} \left\{ \frac{\dot{\chi}^2}{2} + \frac{(\nabla\chi)^2}{2} + 2\,(2|v|-1)\,\chi^2 \right\} \\
E_{\phi_i} &= \sum_{sites} \left\{ \frac{\dot{\phi_i}^2}{2} + \frac{(\nabla\phi_i)^2}{2} + 2\,(|v|-1)\,\phi^2 \right\} \qquad i = 2,3,4 \\
E_{int} &= \sum_{sites} \frac{4}{3} \left\{ \left[(v+\chi)^2 + \vec{\phi}^2\right]^{3/2} - |v|^3 \left[1 + 3\frac{\chi}{v} + 3\frac{\chi^2}{v^2} + \frac{3}{2}\frac{\vec{\phi}^2}{v^2} \right] \right\} \qquad (3.5)
\end{aligned}$$

The sum of these terms gives the total energy $E_{tot} = \sum_{sites} H$, where H is the Hamiltonian defined in eq.(2.11).

In Fig.10 we plot the evolution of each energy term, normalized to the total energy, for $\delta\dot{\phi}_\alpha^2 = 0.01$ (a), 0.1 (b), 1 (c). As already mentioned, we notice that between $t \sim 3fm$ and $t \sim 7fm$ the pion fields, and expecially the π_2, are amplified whilst the σ energy gets reduced. Furthermore, at $t \sim 10fm$ the interaction term strongly reduces, and the fields practically decouple (even if a small interaction term between σ and π is still present). From the plots of Fig.10, it is evident that increasing the initial energy via the velocity variance leads to an increase of the pions background, whereas it reduces the amplification factor of the pions energy. Again this effect can be explained as due to the action of the incoherent field fluctuations induced by the random initial velocities which oppose themselves to the coherence of the DCC phenomenon. A quantitative computation of the mean value of the energy of the π_2 field for times $t \gtrsim 10fm$, in Fig.10, gives $E_{\pi_2} \simeq 215N\ MeV$ (a), $225N\ MeV$ (b), $440N\ MeV$ (c). where N is the number of fermion species. We repeat, however, that quantitative predictions are beyond the scope

of this simulation. Furthermore the model, besides the limitation to be bidimensional in the space coordinates, is characterized by an undetermined number of fermion species N. Of course it would be interesting above all to evaluate the number of produced pions, which in principle could be done for instance in the framework of coherent states.

Source dimension

In this subsection we want to discuss the role of the size of the source. We notice that it may be plausible that there exists a critical volume (a surface in our simulation) defining a lower size for the source, below which no macroscopic correlation phenomena should take place. This can be qualitatively argued following ref.[7]. Let us consider the Fourier transform of the equation of motion that we derive from the Hamiltonian (2.11), where we have approximated the nonlinear term by its spatial average $\langle|\phi|\rangle(\tau)$

$$\frac{d^2}{d\tau^2}\tilde{\phi}_\alpha(\vec{k},\tau) = \left[-\vec{k}^2 \ + \ 4 \ - \ 4\langle|\phi|\rangle(\tau)\right]\tilde{\phi}_\alpha(\vec{k},\tau) \ - \ \lambda\delta_{\alpha 1} \tag{3.6}$$

At the beginning of the expansion, when $\langle|\phi|\rangle \simeq 0$, the enhanced modes are those with $k < k_{max} \simeq 2$ (in dimensionless units), which correspond to a minimal wavelength of about $2fm$. Anyway, if the linear dimensions of the source are less than some scale of an order of magnitude determined by $k_{max}(t)$, the growth of long-wavelength modes cannot take place. We have verified this for a source of radius $0.5fm$: no correlated regions form.

The case of annealing

Finally, we show a sketch of the time evolution of a different initial configuration in Fig.11. Here ϕ_α are initially located at the absolute minimum everywhere (annealing), whereas an energetic source (of 1 fm radius) is still present due to the spatial modulation of the velocity variance. This is done with an exponentially decreasing function whose tail now extends over the whole lattice [10]. We show these pictures to stress that the formation and growth of R-correlated regions can be present without any θ disorientation, only coming from the spatial distribution of the fluctuation energy. In this case the phenomenon of growth proceeds by following a process analogous to spinodal decomposition in alloys, since the equality of the areas comes out to be approximately conserved in time. Anyway this phenomenon occurs in regions where the energy density is very low, and thus it is not relevant to produce correlated pions. Furthermore it is washed out by the expansion of the source, for later times: this is due to the fact that the source carries out most of the energy, which is not spatially ordered. In conclusion the relevant features appear to be those related to the modeling of the central part of the source, rather than to the other details, such as the tail of the energy decay.

4. Conclusions and Outlook

In our simulation we have adopted initial conditions which allow to control the energy of the system, and we have employed fine lattice spacing. Among the quantities we have computed there are the θ angle between the σ and π fields, the local ratios $R_a = \pi_a^2/\vec{\pi}^2$ ($a = 1, 2, 3$), the quadratic term of the energies of the fields, and the higher order term, for representative times. Of course all of them are intimately related, but there is no redundancy in the description as some of them appear to be the more appropriate ones according to the stage of the expansion. Actually we remark that during the first time of the expansion the σ and $\vec{\pi}$ fields are strongly coupled and thus the language in terms of asymptotic particles can be misleading. On the contrary quantities related to the $O(4)$ angles are always well defined and the picture emerging is clarified by the time evolution of the whole set of the parameters.

The θ angle drives the formation of DCC, whilst the crucial feature we look for is the coherence of the pion field oscillations in its internal $O(3)$ space, which is rather signaled by the formation and growth of R_a-s correlated regions. Nevertheless these features are linked, since from our numerical simulation it is clearly confirmed that the formation and growth of R_a-s correlated regions is a macroscopic effect only when the θ angle undergoes relevant and coherent disorientations during the expansion. Actually the phenomenon of growth appears to be driven by the energy excess at different points which tends to align the pion field along a same direction. Now, if the dynamics produces a strong and coherent θ fluctuation, a relevant part of the σ-field energy is transferred to the pion field. This happens in one of the equally-likely $O(3)$ directions and generally manifests itself in a big energy excess over long wavelengths, which is able to produce a macroscopic effect on the growth phenomenon in terms of the R_a's. We recall that the σ and $\vec{\pi}$ fields have to be nearly degenerate only in the central part of the source, whereas most of the initial energy is carried by the σ field.

We also show that phenomena of growth can be present even for very small θ disorientation, due to a non uniform spatial energy distribution over a certain scale. Anyway, in this case, the effect is much less important and furthermore model-dependent.

For large enough times, when the interaction energy is small (for instance, with a source of radius 1 fm, it is enough to have $t \gtrsim 10\ fm$), all the fields are almost decoupled, their energies are separately conserved by the dynamics, and one can talk of particles energy and number.

The indication we get from this study is that the relevant feature to get large R-correlated regions (and thus also to produce an unusual pion distribution), is the initial

energy release in the "hot" vacuum and its relative weight with respect to that at the boundary (which is related to the dimension of the source and to the shape of the energy decay out of the source). We also find that, under these conditions, an increase in the initial energy density leads generally to a reduction of the relative energy enhancement of the pion fields.

The nice feature resulting from this study is that we get a clear description of the DCC mechanism by means of $O(4)$-angles, and that we can allow for a complete description of the final-states. Although still not apt to give physical quantitative results because of its low dimensionality, our work offers a guide to the exploration of the driving features of the mechanism itself. Among them, our simulation confirms that the role of random fluctuations is to disturb the formation of large regions of coherence, whereas it further suggests that the size of the "hot" source has to be large enough in order to produce a macroscopic effect. Following these indications, a large amount of energy release, compatible with a large enough number of produced pions and with small field fluctuations, can be obtained only by increasing the source size. This could hopefully occur if the expanding fireball cools near T_c having reached a linear size of at least some fm. Other details, such as the shape of the energy decay, look less important.

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Appendix

We summarize briefly the algorithm used in the numerical integrations. We have always used a second order algorithm, belonging to the class of the so-called "bilateral algorithm" (see ref. [18]); it has the remarkable property of being symplectic, i.e. it performs a canonical transformation at every step of integration, thus faithfully preserving the qualitative structure of the dynamics. The simplecticity also ensures a very good energy conservation, while the use of a second order algorithm allows for the use of comparatively long integration steps (a second order algorithm is one for which the error introduced at each step is $\mathsf{O}(\Delta t^3)$) without precision-loss, hence reducing the cpu-time needed for the simulation. The explicit scheme of the algorithm used is the following:

$$\begin{cases} \tilde{q}_i = q_i(t) \\ \tilde{p}_i = p_i(t) + \frac{1}{2}\Delta t\, f_i[q(t)] \\ q_i(t+\Delta t) = \tilde{q}_i + \Delta t\, \tilde{p}_i \\ p_i(t+\Delta t) = \tilde{p}_i + \frac{1}{2}\Delta t\, f_i[q(t+\Delta t)] \\ \hat{p}_i = p_i(t+\Delta t) \\ \hat{q}_i = q_i(t+\Delta t) + \frac{1}{2}\Delta t\, \hat{p}_i \\ p_i(t+2\Delta t) = \hat{p}_i + \Delta t\, f_i(\hat{q}_i) \\ q_i(t+2\Delta t) = \hat{q}_i + \frac{1}{2}\Delta t\, p_i(t+2\Delta t). \end{cases} \tag{A1}$$

(Here f_i are the forces. $\tilde{q}_i, \tilde{p}_i = p_i, \hat{q}_i, \hat{p}_i$ are dummy variables lacking of any physical meaning). The scheme is called bilateral because it performs two integration steps at a time by the rôle of the q and p. This feature results in an enhancement of the precision with which energy is conserved. In typical simulations, we found that energy fluctuated around its mean value with a relative amplitude of about 10^{-5} having set the time step to 0.01. It is also remarkable that the mean value of these fluctuations vanishes: this ensures that the system is not subject to spurious damping or excitations. (For further details of the method see ref.[18]).

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Figures

Fig.1 Plot of the effective potential of eq.(2.7) vs. $\phi_1 \propto \sigma$, at $\vec{\pi} = 0$ (the vacuum energy density has been subtracted).

Fig.2 Plot of the $\phi_1 \propto \sigma$ configuration on the lattice at $t = 0.02fm$ ($\delta\dot{\phi}_\alpha^2 = 0.1$).

Fig.3 Plot of the $\phi_3 \propto \pi_2$ configuration on the lattice at $t = 0.02fm$ ($\delta\dot{\phi}_\alpha^2 = 0.1$).

Fig.4 Plot of the energy density shape at $t = 0.02fm$, on the internal 50×50 lattice centered in the source (see eq.(2.11). Here $\delta\dot{\phi}_\alpha^2 = 0.1$.

Fig.5 Plots of the θ angle on the internal 32×32 square lattice centered in the source (initially the source radius is $\rho \simeq 10$ sites, Fig.(a)), for representative times ($\delta\dot{\phi}_\alpha^2 = 0.1$). The range $[0, \pi]$ has been divided in five equally spaced regions colored according to a gray scale, ranging from black ($\theta \simeq 0$) to white ($\theta \simeq \pi$). These figures explicitly show three different behaviours. At early times there is a formation and growth of θ correlated regions inside the source. Such regions tend to melt together in the bulk, while the outer regions align to $\theta = \pi$. This phenomenon ends up at about $t = 1fm$, when the whole source is aligned with the true vacuum. Anyway, the system has not relaxed to the true vacuum yet. From the center of the source, in fact, a collective motion starts, which brings, at $t \simeq 2.5fm$, the whole source to be at $\theta = 0$. Afterwards, the system begins to perform damped oscillations along the bottom of the potential well.

Fig.6 In Fig.(a) we plot $\langle\theta(r)\rangle(t)$. The data refer to the same simulation as in Fig.5. The mean value is taken on concentric rings of thickness $0.4fm$. This picture points out the time scales characterizing the different stages of the phenomenon of DCC. In Fig.(b) we plot the shape of two sections at fixed radius $\bar{r}$, with $0 < \bar{r} < 0.4\ fm$ (continuous) and $0.8 < \bar{r} < 1.2\ fm$ (dashed).

Fig.7 In these figures the θ configurations on the (x, y) lattice is given by the arrow orientations. These pictures are taken in coincidence of some maxima of disorientation as it can be seen in Fig.6 (they refer to the same simulation). As in Fig.5, it can be noticed that the disorientation is spatially incoherent at $t = 0.02fm$ (Fig.(a)), whereas it has become coherent at $t = 2fm$ and $t = 5fm$ (Fig.(b)-(c)). Finally, in Fig.(d), at $t = 9fm$, the presence of concentric regions of different θ orientation is evident. To get a satisfying resolution in Fig.(a) and (b) only the central 25×25 lattice has been plotted (outside $\theta = \pi$), whereas the central 50×50 in Fig. (c) and 100×100 lattice in Fig.(d) have been taken into account. Then the average over 2×2 (Fig.(c)) and 4×4 blocks (Fig.(c)) has been performed.

Fig.8 Distributions of the representative points of the lattice sites in the (φ, η) plane (see eq.(3.3)). The data refer to the same simulation as in the previous pictures. At the beginning of the simulation all the directions are equally likely and thus the distribution is uniform. Then the existence of preferred directions in which the pion field oscillates starts to be evident (from $t \simeq 4fm$) and becomes more evident for later times. The residual noise which is still visible for $t = 12fm$ in Fig.(d) is mostly due to the points of the lattice which are not yet reached from the expanding source.

Fig.9 Configuration of the ratio R_2 ($R_a \equiv \pi_a^2/\vec{\pi}^2$) at each lattice site for representative times ($\delta\dot{\phi}_\alpha^2 = 0.1$). To get a pictorial representation, the interval of definition of R_a has been divided in three regions which are initially equally probable: black if $R_a \in [0, 1/9]$, grey if $R_a \in [1/9, 4/9]$ and white if $R_a \in [4/9, 1]$. These pictures show the formation of large R_2-correlated regions in which the pion field oscillates along a preferred direction (see also Fig.8).

Fig.10 Plots of the quadratic terms of the energy of each field and of the higher order term (see. eq.(3.5)), normalized to the total energy, vs. time. With reference to eq.(3.5), the curves give the behaviour respectively of: E_χ (continuous), E_{ϕ_2} (dotted-dashed), E_{ϕ_3} (dashed), E_{ϕ_4} (dotted-long dashed), E_{int} (dotted). We remember that $(\phi_2, \phi_3, \phi_4) \propto (\pi_1, \pi_2, \pi_3)$.

Notice that between $t \simeq 3fm$ and $t \simeq 7fm$ the pion fields, and expecially the π_2 field, are amplified. Furthermore, at $t = 10fm$ the interaction term strongly reduces, and each field behave almost as free. It is also evident that the increase of the initial velocity variance (from Fig.(a) to Fig.(c)) leads to an increase of the pions background, whereas it reduces their energy amplification.

Fig.11 Plot of the ratio R_2 in the case in which the system is initially set at the bottom of the potential well of Fig.1. The initial velocity variance has been modulated with an exponentially decreasing function whose tail extends over the whole lattice [10]. These figures show that the formation and growth of R-correlated regions can be present even without any θ disorientation, due only to the spatial distribution of the energy. Anyway this phenomenon occurs in regions where the energy density is very low, and thus it is not relevant to produce correlated pions. Furthermore it is washed out by the expansion of the source, for later times.